

TECHNICAL MEMORANDUMS

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

No. 958

GENERAL RELATIONSHIPS BETWEEN THE VARIOUS SYSTEMS  
OF REFERENCE AXES EMPLOYED IN FLIGHT MECHANICS

By H. J. Rautenberg

Luftfahrtforschung  
Vol. 17, No. 4, April 20, 1940  
Verlag von R. Oldenbourg, München und Berlin

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November 1940

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OF REFERENCE AXES EMPLOYED IN FLIGHT MECHANICS\*

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SUMMARY

The different possibilities of orientation of the systems of axes currently employed in flight mechanics are compiled and described. Of the three possible couplings between the wind and aircraft axes, the most suitable coupling is that in which the  $y$  axis is made the principal axis of rotation for one of the two coupling angles (angle of attack  $\alpha$ ). This coupling is termed the "E coupling."

In connection with this coupling, an experimental system of axes is introduced, whose axes  $x_0$  and  $z_0$  are situated in the plane of symmetry of the airplane and rotate about the airplane lateral axis  $y = y_0$ . This system of axes enables the utilization of the coefficients obtained in the wind tunnel in the flight-mechanic equations by a simple transformation, with the aid of the angle of attack  $\alpha$ , measured in the plane of symmetry of the airplane.

With the introduction of a third coupling angle, the E coupling is extended to the case of the airplane on a ground plate. A special "system of wind axes to be used for measurements with ground plate" is explained.

Of three possible couplings between the aircraft axes  $x, y, z$  or the wind axes  $x_a, y_a, z_a$  and the ground axes  $x_g, y_g, z_g$ , the most suitable coupling is obtained if the  $z$  axis of the basic ground system and the  $x$  axis of either the aircraft or the wind system, respectively, are made the principal axes of rotation.

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\* "Die gegenseitige Kopplung der flugmechanischen Achsenkreuze." Luftfahrtforschung, vol. 17, no. 4, April 20, 1940, pp. 106-122.

## I. INTRODUCTION

In the treatment of flight mechanic problems it is always necessary to employ various systems of axes. Now, there are different ways of introducing these axes and particularly, their orientation. The definitions of the reference axes published in national and international literature, their sense of positive direction, and their mutual angle of reference are, in part, much at variance; so that in many cases it is impossible to make a comparison of test data without first effecting more or less tedious conversions.

Attempts have therefore been made from time to time to set up a standard system of definitions among the various countries, for the most important systems of axes and angles of reference. The discussions regarding such standardization so far, have always been hampered by the necessity of preparing a complete list of all reference axes and mutual reference angles, showing their advantages and disadvantages when applied to flight mechanics. Such a compilation of the possible and practical reference axes and reference angles has, however, never been available.

In the present report, the various possibilities of coupling the individual axes are discussed, and their practicability critically analyzed. A standard system of reference axes and angles is developed. Compliance with a few fundamental requirements makes a logical development of this standard system possible, which in no wise is in contradiction with the requirements of practical applications.

The general basic laws applied in the orientation of two spatial systems of axes are represented in a form discussed in reference 9, which makes it possible to apply these laws in a comprehensive manner to the systems of axes needed in flight mechanics and their mutual coupling.

The results of the present study assume that the reader is familiar with the cited report (reference 9).

## II. GENERAL ASSUMPTIONS

One of the most essential points in the development of a system of reference axes, is an appropriate defini-

tion of the reference or coupling angles. The determination of the positive sense of direction of the individual reference axes is far less important than a suitable disposition of the coupling angles.

All coupling angles of the flight-mechanic axes shall satisfy the following basic requirements:

1. The individual coupling angles must be unrelated:

This requirement is met when three Euler angles tie two systems of axes together. For exact definition of these angles, see reference 9, which also contains a detailed description of the model representation of the Euler angles by a Cardan system.

2. A reference angle is positive when the rotation about the axis of the particular angle follows the direction of the positive axis of rotation, viewed clockwise (right rotation). By "axis of rotation" is meant the axis about which the rotation occurs, which is measured by the pertinent angle.

For the sake of clarity, we shall always give, apart from the definitions, the axes of rotation of the angles, as well as the planes in which the angles are measured.

As a general rule, the axes of rotation of the Euler angles which orientate the two systems of axes  $K_1$  and  $K_2$ , follow the law:

The axis of rotation of one of the three coupling angles is an axis of the system  $K_1$ , the axis of rotation of a second coupling angle is an axis of system  $K_2$ , and the axis of rotation of the third coupling angle is the junction line between  $K_1$  and  $K_2$  (fig. 1).

The two systems of axes are, for the present, assumed to be in agreement, so that the corresponding axes are coincident ( $x_1$  with  $x_2$ ,  $y_1$  with  $y_2$ ,  $z_1$  with  $z_2$ ). To analyze a certain mutual position of the two systems, one of the systems (the system to be orientated) is turned from the neutral position relative to the assumedly restrained second system and termed the "directional system." If this rotation is effected in the positive sense about the previously established axes of rotation, the angles of reference are, by definition, positive. The axis of the orientating or directional system of axes forming the axis

of rotation for one of the Euler angles is termed the "principal axis of rotation" of the orientating system, while the axis of the system to be orientated relative to the orientating system, which is the axis of rotation for one of the Euler angles, is called the "definitive axis of rotation" of the axes to be orientated.

It follows from the report (reference 9) that the coupling of one system with a second through three Euler angles, can be effected in more than one way; for after determination of one of the systems to be orientated as the orientating system and its principal axis of rotation, it affords three different possibilities of defining the angles of reference, depending upon the selection of one of the other three axes as definitive axis of rotation. The angles in all these three cases are Euler angles.

The immediate problem is to establish the best practical coupling between the individual systems of axes. This includes:

- 1) the coupling between airplane and wind axes;
- 2) the coupling between the wind and ground axes;
- 3) the coupling between the airplane and ground axes.

The mutual coupling of the individual systems of axes is to conform to the following rules:

- 1) The ground system is, in every case, the orientating system; both the airplane and wind axes rotate relative to the assumedly fixed ground axes.
- 2) In the coupling between airplane and wind axes, the latter shall constitute the wind axes of the orientating system. Thus, the airplane axes rotate with respect to the assumedly fixed wind axes.

### III. COUPLING OF AIRPLANE AND WIND AXES

In the orientation of a system of airplane axes  $x$ ,  $y$ ,  $z$  relative to a wind-axes system  $x_a$ ,  $y_a$ ,  $z_a^*$ , the report (reference 9) cites a specific case where one of

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\*These axes are defined in reference 10.

the three reference angles is always zero. Since at first only one single axis - the relative wind axis  $x_a^*$  - of a wind-axes system is physically given, the orientation of the airplane and wind axes is effected by two variable angles - so-called "aerodynamical angles" - while the third angle necessary to a complete orientation, is zero. It has been shown (reference 9) that the Euler angle, which as axis of rotation possesses the only originally given axis of the still incomplete system or orientating axes, is zero. The introduction of axes  $y_a$  and  $z_a$ , with which the wind-axes system becomes complete, is necessary for the presentation of the forces and moments. The general laws are now to be applied to the case in point.

As orientating-axes system, the wind axes of which, however, only  $x_a$  is given so far, are chosen. The sole completely known axis must be chosen as principal axis of rotation. Of  $y_a$  and  $z_a$ , it is merely assumed that they are at right angles to each other and to  $x_a$ , and that they form a right-hand system of axes with  $x_a$ . To define the position of  $y_a$  and  $z_a$ , one of them must be tied to the aircraft system  $x, y, z$  by a specified order. But then a rotation of axes  $x, y, z$  relative to the axes  $x_a, y_a, z_a$  about the principal axis of rotation  $x_a$  becomes impossible. Hence the angle of the principal axis of rotation is zero and the complete orientation between  $x_a, y_a, z_a$  and  $x, y, z$  by two angles can be achieved. These conditions can equally be explained by stating that the given wind axis  $x_a$  is directed to the aircraft axes, for which two angles are sufficient.

When stating that, by tying one of the axes  $y_a$  and  $z_a$  to the airplane axes, no rotation of the aircraft-axes system with respect to  $x_a, y_a, z_a$  about the  $x_a$  axis, is possible, it must not be confused with the obvious fact that the total Cardan system - consisting of  $x, y, z$  and  $x_a, y_a, z_a$  can be arbitrarily rotated with respect to a system of space axes. Physically, this fact implies that the flow condition described by the two aerodynamic angles is not changed by a rotation of the airplane (nor hence of the wind axes tied to the airplane) about the wind axis.

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\* By "wind axis" is meant an axis in the direction of flow on the airplane, assumed as parallel flow. In general, it may be presumed that the wind axis is practically coincident with the tangent of the spatial flight path.

On the Cardan model (fig. 1), the two angles which direct the aircraft axes to the wind axes, can be described by the rotation of the inner ring  $R_2$  about axis  $A_2$  (junction line), and by the rotation of body  $K$  (the airplane in this case) about its definitive axis of rotation  $AK$ . Hereby the neutral position of ring  $R_1$  changes by equal position of the airplane in space, depending on the choice of one of the three aircraft axes as definitive axis of rotation.

The only plane of the wind axes given at start by the wind axis, is the plane of the cross-wind force  $y_a z_a$  at right angles to the wind axis. The position of the rectangular plane axes  $y_a, z_a$  in this plane is, for the present, undetermined. The intersection of the plane  $y_a z_a$  and planes  $xy, yz$ , and  $zx$  of the aircraft system furnishes a straight line, which for each of the three cases defines the position of one of the two axes at right angles to  $x_a$ . These three potential intersections correspond to three possibilities of selecting one each of the three aircraft axes as definitive axis of rotation (figs. 2, 3, and 5). The definitive axis of rotation is always at right angles to the contemporary straight line and carries the notation which is not contained in the notation of this body plane at intersection of plane  $y_a z_a$  with one of the three body planes. The straight line itself is identical with the junction line and represented by axis  $A_2$  (fig. 2) in the Cardan model. The three potential couplings are appended in table I.

TABLE I

Designation	Intersection of plane $y_a z_a$ with	Axis defined by the straight line	Definitive body axis	Position of Cardan rings for neutral setting of axis
F coupling	$xy$	$y_a$	$z$	Crossed
E coupling	$zx$	$z_a$	$y$	Crossed
Not named	$yz$	$x_a$	$x$	Parallel

Our next attempt is to analyze and disprove the practical use of the third possibility for flight-mechanic application.

On studying the neutral setting in which the  $x$  axis of the airplane coincides with the wind axis  $x_a$ , it is seen that the plane  $y_a z_a$  intersects only the two planes  $xy$  and  $zx$ , while coinciding with plane  $yz$ .

In this instance there is no straight line between plane  $y_a z_a$  and  $yz$ , and the position of one of the axes  $y_a$  and  $z_a$  cannot be definitely established, hence remains undetermined. If the body  $x$  axis does not coincide with the wind axis  $x_a$ , the two planes  $y_a z_a$  and  $yz$  meet. The straight line then defines the crosswind axis  $y_a$ . The definitive body axis, which is always at right angles to the straight line, is hereby the  $x$  axis. Now, it is quite possible that, in an analysis of airplane motion with respect to the surrounding air, under normal flow conditions, the two systems of axes are in neutral setting to each other where the body  $x$  axis coincides with the wind axis  $x_a$ . On the Cardan system (fig. 2), the picture is as follows: The axes of rotation for the two reference angles are  $A_2 = y_a$ , and the definitive body axis  $AX = x$ . Axis  $A$  of the Cardan model represents the wind axis  $x_a$ . In neutral setting the inner ring  $R_2$  is parallel with the outer ring  $R_1$ . Setting an angle of autorotation by a rotation about the definitive axis  $x$  while the angle, which measures a rotation about  $A_2$ , is to remain zero, this autorotation angle (angle of roll) does not at all describe the position of the Cardan axis  $A_1$  (wind axis  $x_a$ ) relative to the aircraft system, but it describes a rotation about the Cardan axis  $A_1$  (wind axis  $x_a$ ) with which the body  $x$  axis coincides. Viewed physically, any rotation of the body axes about axis  $x_a$  can take place without modifying the state of flow relative to the airplane. The autorotation angle can therefore by accord of wind axis  $x_a$  with body axis  $x$ , assume any value without changing the state of flow. In other words, it is unsuitable for coupling the body axes to the wind axes.

It is emphasized that, for instance, in the choice of the principal axes of inertia as body axes, the case cited here - where the  $x$  axis coincides with the  $x_a$  axis - can occur at practically any time under normal flight positions and conditions of flow.



## F Coupling between Aircraft and Wind Axes

The intersection of the plane  $y_a z_a$  at right angles to the wind axis with the plane  $xy$  of the airplane, manifests that the straight line between the two planes definitely defines the cross-wind axis  $y_a$  and so ties one of the two axes at right angles to wind axis  $x_a$  in a well-defined manner to the airplane axes, for  $y_a$  always in the  $xy$  plane of the airplane axes. With the thus-given position of  $x_a$  and  $y_a$  the third, or lift axis  $z_a$ , is also ascertained. It is always situated in the plane  $z x_a$ . The sense of direction of the positive wind axes shall be defined conformably to the arguments advanced in reference 9, which are as follows: The  $x_a$  axis opposite to the stream direction of the relative wind, is positive. The positive  $y_a$  axis, viewed opposite to the relative wind, points in normal flight to the right; the positive  $z_a$  axis, under the same conditions, downward.

For this particular coupling of body axes and wind axes, termed the "F coupling" and characterized by the definition of the cross-wind axis as a trace between the planes  $xy$  and  $y_a z_a$ , the  $z$  axis represents the definitive axis of rotation of the aircraft axes.

The two coupling angles of the F coupling between aircraft and wind system are explained in

TABLE II

Angle	Symbol	Definition of angle	Test plane of angle	Axis of rotation of angle
Angle of attack	$\alpha_F$	$xy$ plane against $x_a$ or $z$ against $z_a$	$z_a x_a$	Junction line, i.e., $y_a$ axis
Angle of yaw	$\beta_F$	$x$ against $z_a x_a$ plane or $y$ against $y_a$	$xy$	Definitive body axis, i.e., $z$ axis

The trace between the planes  $y_a z_a$  and  $xy$  is no longer defined at  $\alpha_F = 90^\circ$ , because both planes then coincide. In this case the position of  $y_a$  and  $z_a$  becomes undetermined.

The F coupling is illustrated on the Cardan model, figure 3.

The two aerodynamic angles  $\alpha_F$  and  $\beta_F$  can also be very clearly defined with the velocity component  $v_x$ ,  $v_y$ ,  $v_z$  of the airplane in the direction of the body axes and the resultant flight speed  $v_{x_a}$ . The following relations are applicable:

$$\sin \alpha_F = \frac{v_z}{v_{x_a}} \quad (I)$$

$$\cos \alpha_F = \frac{\sqrt{v_x^2 + v_y^2}}{v_{x_a}} \quad (II)$$

$$\tan \alpha_F = \frac{v_z}{\sqrt{v_x^2 + v_y^2}} \quad (III)$$

$$\sin \beta_F = \frac{v_y}{v_{x_a} \cos \alpha_F} \quad (IV)$$

$$\cos \beta_F = \frac{v_x}{v_{x_a} \cos \alpha_F} \quad (V)$$

$$\tan \beta_F = \frac{v_y}{v_x} \quad (VI)$$

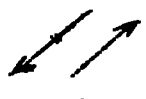
The tie-up between the direction cosines and the reference angles  $\alpha_F$  and  $\beta_F$  that orientate the body axes to the wind axes, is traced with the aid of figure 4.

The application of the cosine law yields:

- From triangle  $x x_a P$ : 1.  $\cos (x x_a) = \cos \alpha_F \cos \beta_F$   
 2.  $\cos (x y_a) = \sin \beta_F$
- From triangle  $x z_a P$ : 3.  $\cos (x z_a) = -\sin \alpha_F \cos \beta_F$
- From triangle  $y x_a P$ : 4.  $\cos (y x_a) = -\cos \alpha_F \sin \beta_F$   
 5.  $\cos (y y_a) = \cos \beta_F$
- From triangle  $y z_a P$ : 6.  $\cos (y z_a) = \sin \alpha_F \sin \beta_F$   
 7.  $\cos (z x_a) = \sin \alpha_F$
- From triangle  $z y_a P$ : 8.  $\cos (z y_a) = 0$   
 9.  $\cos (z z_a) = \cos \alpha_F$

The transfer from aircraft axes to wind axes, based on the F coupling, is shown in

TABLE III

		Wind Axes		
		$x_a$	$y_{a_F}$	$z_{a_F}$
Aircraft or body axes	x	$+\cos \alpha_F \cos \beta_F$	$+\sin \beta_F$	$-\sin \alpha_F \cos \beta_F$
	y	$-\cos \alpha_F \sin \beta_F$	$+\cos \beta_F$	$+\sin \alpha_F \sin \beta_F$
	z	$+\sin \alpha_F$	0	$+\cos \alpha_F$

(Refer to fig. 4)

It is again pointed out that the axes  $y_a$  and  $z_a$  are differently defined for each of the two couplings F and E, and therefore do not agree; when in an analysis of a certain flow attitude, for example, first the F coupling, and then the E coupling is used as basis. The wind axis  $x_a$  is unaffected by the choice of coupling. Hence, in comparative studies of E and F couplings, axes  $y_a$  and  $z_a$  must also carry the subscripts E and F, as well as the aerodynamic angles  $\alpha$  and  $\beta$  (fig. 4, table III).

### E Coupling between Aircraft Axes and Wind Axes

The intersection of the cross-wind plane  $y_a z_a$  with the  $zx$  plane of the body axes, defines an axis at right angles to the body axis, which is always in the plane of symmetry  $zx$  of the airplane. Since the lift is recorded along this axis, it is called lift axis. With the position of the wind axis and of the lift axis, the position of the cross-wind axis at right angles to the other two, is itself defined. The positive direction of the axis is as stated on page 8.

This coupling between body axes and wind axes is termed the "E coupling." The definitive axis of rotation is the  $y$  axis. The two reference angles between body axes and wind axes are explained in

TABLE IV

Angle	Symbol	Definition of angle	Test plane of angle	Axis of rotation of angle
Angle of attack	$\alpha_E$	$x$ against $x_a y_a$ plane or $z$ against $z_a$	$zx$	Definitive body axis, i.e., $y$ axis
Angle of yaw	$\beta_E$	$zx$ plane against $x_a$ or $y$ against $y_a$	$x_a y_a$	Junction line, i.e., $z_a$ axis

The trace between plane  $y_a z_a$  and plane  $zx$ , which defines the lift axis, is no longer defined at  $\beta_E = 90^\circ$ , thus leaving the position of  $y_a$  and  $z_a$  in this case undetermined.

Figure 5 shows the Cardan model for the E coupling.

The two aerodynamic angles  $\alpha_E$  and  $\beta_E$  can again be defined with the help of the velocity components  $v_x$ ,  $v_y$ ,  $v_z$  of the airplane and the resultant flight speed  $v_{x_a}$ . The following formulas are applicable:

$$\sin \alpha_E = \frac{v_z}{v_{x_a} \cos \beta_E} \quad (\text{VII})$$

$$\cos \alpha_E = \frac{v_x}{v_{x_a} \cos \beta_E} \quad (\text{VIII})$$

$$\tan \alpha_E = \frac{v_z}{v_x} \quad (\text{IX})$$

$$\sin \beta_E = \frac{v_y}{v_{x_a}} \quad (\text{X})$$

$$\cos \beta_E = \frac{\sqrt{v_x^2 + v_z^2}}{v_{x_a}} \quad (\text{XI})$$

$$\tan \beta_E = \frac{v_y}{\sqrt{v_x^2 + v_z^2}} \quad (\text{XII})$$



The tie-up between the direction cosines and coupling angles  $\alpha_E$  and  $\beta_E$  is now deduced from figure 6:

The following relations result:

- From triangle  $x x_a P$ : 1.  $\cos (x x_a) = \cos \alpha_E \cos \beta_E$   
 From triangle  $x y_a P$ : 2.  $\cos (x y_a) = \sin \beta_E \cos \alpha_E$   
 3.  $\cos (x z_a) = -\sin \alpha_E$   
 4.  $\cos (y x_a) = -\sin \beta_E$   
 5.  $\cos (y y_a) = \cos \beta_E$   
 From triangle  $y z_a P$ : 6.  $\cos (y z_a) = 0$   
 From triangle  $z x_a P$ : 7.  $\cos (z x_a) = \sin \alpha_E \cos \beta_E$   
 From triangle  $z y_a P$ : 8.  $\cos (z y_a) = \sin \alpha_E \sin \beta_E$   
 9.  $\cos (z z_a) = \cos \alpha_E$

The transfer from body axes to wind axes, based on the E coupling, is indicated in table V.

TABLE V

 Aircraft or body axes		W i n d   A x e s		
		$x_E$	$y_{aE}$	$z_{aE}$
 Aircraft or body axes	x	$+\cos \alpha_E \cos \beta_E$	$\cos \alpha_E \sin \beta_E$	$-\sin \alpha_E$
	y	$-\sin \beta_E$	$+\cos \beta_E$	0
	z	$+\sin \alpha_E \cos \beta_E$	$+\sin \alpha_E \sin \beta_E$	$+\cos \alpha_E$

A comparison with table III shows that table V is obtained by reflection of the separate expressions on the dashed diagonal of table III. The prefixes of the reflected expressions are reversed.

### The Experimental System of Axes

The E coupling between body axes and wind axes leads to a further system of axes whose significance is fully proved later on.

Since angle  $\alpha_E$  in the E coupling has the y axis of the body system as axis of rotation, and is therefore measured in the plane of symmetry of the airplane, the axes  $x_e$ ,  $y_e$ ,  $z_e$  can be defined as follows:

1) The trace of the intersection of the plane of symmetry with the plane  $x_a y_{aE}$  gives an axis  $x_e = OP$  (fig. 6). This axis  $x_e$  is therefore always in the plane of symmetry  $zx$  of the airplane (fig. 7).

2) Since  $x_e$  lies in the plane of symmetry - that is, at right angles to the body lateral axis y - the body lateral axis is a further axis of this system of axes; hence,  $y_e = y$ .

3) The third axis  $z_e$ , which must be at right angles to  $x_e$  and  $y_e$ , therefore lies also in the plane of symmetry of the airplane. The position of  $z_e$  is likewise defined by the trace passing through the plane of symmetry and the cross-wind plane  $y_{aE} z_{aE}$ . Since, in addition, this trace defines the lift axis  $z_{aE}$  itself, we

have  $z_e = z_{aE}$ . The  $x_e y_e z_e$  axes are called "experimental axes" because of their general use for presentation of wind-tunnel data. Axes  $x_e$  and  $z_e$  form a system of axes bound to the plane of symmetry, which can rotate about  $y$  in the plane of symmetry.

Axis  $x_e$  can also be explained as follows: First,  $x_e$  results when the body axis  $x$  inclined at angle  $\alpha_E$  relative to plane  $x_a y_{aE}$  is visualized as being turned back through the very angle  $\alpha_e$  in the plane of symmetry. This backward rotation has the  $y$  axis of the body system as axis of rotation. Secondly, the  $x_e$  axis results from visualizing the wind axis  $x_a$  inclined through angle  $\beta_E$  relative to  $zx$  as being turned back through the same angle  $\beta_E$  in the  $x_a y_{aE}$  plane. This backward rotation has the lift axis  $z_{aE}$  as axis of rotation.

Of the three axes of the experimental system, one refers to the body axes ( $y_e = y$ ), one to the wind axes ( $z_e = z_{aE}$ ), defined by the E coupling, and the third, to the trace between a plane of the body axes ( $zx$ ) and a plane of the wind axes ( $x_a y_a$ ). In addition, the axis of the experimental system related to the wind axes ( $z_e = z_{aE}$ ) is at the same time a trace between a plane ( $zx$ ) and a plane ( $y_{aE} z_{aE}$ ), and the axis of the experimental belonging to the body axes ( $y_e = y$ ) is at the same time the trace between a plane of body axes ( $yz$ ) and a plane of the wind axes ( $x_a y_{aE}$ ).

Because of the described connections, the experimental axes system can be looked upon as a type of body and of wind system. In fact, it represents to a certain extent, a combination of both systems and is an excellent bridge between the body axes  $xyz$  and the wind axes  $x_a y_{aE} z_{aE}$ , defined by the E coupling, which affords a simple way of bridging the two systems. The excellent position of the plane of symmetry of the airplane and the body  $y$  axis at right angles to it, is readily apparent. With the E coupling the plane of symmetry is the natural mediator between the body-axes and the wind-axes systems.

Particular importance is attached to the experimental

axes in wind-tunnel tests. It has already been pointed out (reference 9) that "wind axis" and "tunnel axis" are merely two different terms for one and the same concept, hence  $x_{aK} = x_a$ . With the conventional suspension systems in tunnels, the setting of an angle of yaw is achieved by turning the model about the vertical axis at right angles to the flow axis - which is at the same time, the lift axis. The total balance system turns with it about this vertical axis. The moment reference axes in the great majority of German wind tunnels therefore are -

For the yawing moment: the vertical lift axis  
 $z_{aK} = z_{aE}$ ;

For the rolling moment: an axis in the horizontal drag, lateral force plane  $x_{aK} y_{aK}$ , rotated about angle of yaw  $\beta_E$  with respect to  $x_{aK}$ . This axis is also defined as the trace between the plane of symmetry of the model and the drag, cross-force plane.

For the longitudinal pitching moment: an axis in the horizontal drag, cross-force plane  $x_{aK} y_{aK}$  turned through angle of yaw  $\beta_E$  with respect to  $y_{aK}$ , and identical with the  $y$  axis of the model.

It is readily seen that these moment reference axes are in agreement with the axes  $x_e, y_e, z_e$  of the experimental system and therefore forms an excellent bridge between wind-tunnel measurements and applications in flight mechanics.

It is again pointed out that the experimental system possesses this significance only in combination with the coupling between airplane axes and wind axes, where one of the two aerodynamic angles, namely, the angle of attack  $\alpha_E$  as axis of rotation, has the  $y$  axis at right angles to the plane of symmetry - hence, is measured in the plane of symmetry.

In conclusion, it is noted that the axes  $x_e, y_e, z_e$  agree with those termed "wind axes" in U.S. and British literature.

The connection between the direction cosines and angle



$\alpha_E$  which directs the body axes  $x, y, z$  to the experimental axes  $x_e, y_e, z_e$ , is easily seen in figure 7.

The transfer from airplane axes to experimental axes, obtainable from table V, when putting  $\beta_E = 0$ , has the form indicated in

TABLE VI

Body axes	Experimental Axis			
		$x_e$	$y_e$	$z_e$
	$x$	$\cos \alpha_E$	0	$-\sin \alpha_E$
	$y$	0	1	0
	$z$	$\sin \alpha_E$	0	$\cos \alpha_E$

The connections between the direction cosines and angle  $\beta_E$  orientating the wind axes to the experimental axes, are seen at a glance in figure 11. The transfer table from wind axes to experimental axes, which are also obtainable from table V, after putting  $\alpha_E = 0$ , has the form given in

TABLE VII

Experimental axes	Wind Axes			
	$x_a$	$y_{aE}$	$z_{aE}$	
	$x_e$	$\cos \beta_E$	$\sin \beta_E$	0
	$y_e$	$-\sin \beta_E$	$\cos \beta_E$	0
	$z_e$	0	0	1

#### The Practicability of the E and F Couplings between Aircraft Axes and Wind Axes in the Solution of Flight-Mechanic Problems

An attempt is made to ascertain the superiority of one or the other coupling methods.

In any appraisal of the advantages and disadvantages

of these coupling methods, representation of the aerodynamic principles of a theoretical and recordable nature employed in the treatment of flight-mechanic problems in stability and motion equations, must form the starting point. So from this point of view, it should prove extremely advantageous if the same coupling of angles between aircraft and wind axes can be used as basis. Whether this accord can be achieved without inviting disadvantages of another kind, forms a part of the study.

We proceed from the wind-tunnel test, from which a large share of the aerodynamic data is obtained. In the tunnel the model is directed to the flow by the angles  $\alpha_E$  and  $\beta_E$  of the E coupling. That the E coupling is the natural coupling between axes solidly connected with the model and the flow in wind-tunnel tests, is easily understood. Reference 9 also points out that in the study of a wing model in the wind tunnel, a definite body axis - in this case a model-fixed, longitudinal axis  $x$  and consequently, a definite normal axis  $z$  - is not given, whereas an axis at right angles to the plane of symmetry of the pertinent airfoil parts is always definitely known. This axis is body  $y$  axis (lateral axis). For this reason, the  $y$  axis - the only known axis of a system of body axes from the start - becomes the definitive axis of rotation. The choice of  $y$  axis as definitive axis of rotation indicates, however, (see table I), the use of the angles of the E coupling to direct the body axes to the flow in the tunnel. From it follows the conventional application of the wind-tunnel balances of horizontal wind tunnels, where one vertical axis at right angles to the stream axis, is the axis of rotation for the angle of yaw  $\beta_E$  and so, for the moment reference axes  $x_e, y_e$ . The wind axes  $x_E, y_E, z_E$  defined by the E coupling, are therefore identical with the tunnel axes  $x_{aK}, y_{aK}, z_{aK}$ . With this application of the E coupling, it is very simple to transfer moment coefficients referred to  $e$  axes, to any body axes by means of angle of attack  $\alpha_E$ .  $y$  being equal to  $y_e$ , makes a conversion for the longitudinal moment  $M_e$ , superfluous. The tunnel rolling moment  $L_e$ , and the tunnel yawing moment  $N_e$  are, conformable to table VI, transferred to body axes in the following manner:

$$L_{\text{airplane}} = L_e \cos \alpha_E - N_e \sin \alpha_E \quad (I)$$

$$N_{\text{airplane}} = N_e \cos \alpha_E + L_e \sin \alpha_E \quad (II)$$

Here  $L_{\text{airplane}}$  and  $N_{\text{airplane}}$  denote moments referred to any selected body -  $x$  axis or  $z$  axis, respectively. The angle of attack  $\alpha_E$  is defined as angle between the chosen  $x$  axis, to which  $L_{\text{airplane}}$  is also referred, and the drag, cross-wind plane  $x_A y_A$ .

If the body reference axis for  $\alpha_E$  is other than the body moment reference axis  $x$ , the transfer is easy, as  $\alpha_E$  is measured in the plane of symmetry. To illustrate:

The angle of attack  $\alpha_{E_0}$  is defined as the angle between zero lift axis  $x_0$ , and the drag, cross-wind plane  $x_A y_A$ . The moments  $L_e$ ,  $M_e$ ,  $N_e$  are distributed over a system of body axes  $x$ ,  $y$ ,  $z$ , whose  $x$  axis is given by the line intersecting the plane of symmetry and plane of the wing chord (chord axis), the wing being assumed with zero twist.

The angle of attack  $\alpha_E$  between the body moment reference axis  $x$  (chord axis) and the drag, cross-force plane  $x_A y_A$  is built up of angle  $\alpha_{E_0}$  and the constant angle  $\alpha_K$  between the zero lift axis and the chord axis  $x$ . Hence,

$$\alpha_E = \alpha_{E_0} + \alpha_K$$

Consequently,

$$L_{\text{airplane}} = L_e \cos (\alpha_{E_0} + \alpha_K) - N_e \sin (\alpha_{E_0} + \alpha_K) \quad (\text{III})$$

$$N_{\text{airplane}} = N_e \cos (\alpha_{E_0} + \alpha_K) + L_e \sin (\alpha_{E_0} + \alpha_K) \quad (\text{IV})$$

The transfer from one system of body axes  $x_1, y_1, z_1$  to another,  $x_2, y_2, z_2$ , is readily achieved with the E coupling, since a rotation of the body axes in the plane of symmetry produces a change in  $\alpha_E$ , which more or less consists of the addition of the constant angle  $\alpha_K$  to the angle  $\alpha_{E_1}$  referred to  $x_1$ . Here  $\alpha_K$  is the angle between  $x_2$  and  $x_1$  or between  $z_2$  and  $z_1$ , respectively. These connections are explained in continuation of the above example:

The components  $P_x, P_y, P_z$  of a directed quantity  $P$  (moment, angular velocity, etc.) are first referred to a

system of body axes  $x_1, y_1, z_1$ . Now if they are to be referred to another system  $x_2, y_2, z_2$ , the following conversions apply:

$$P_{x_2} = P_{x_1} \cos \alpha_k - P_{z_1} \sin \alpha_k \quad (V)$$

$$P_{z_2} = P_{z_1} \cos \alpha_k + P_{x_1} \sin \alpha_k \quad (VI)$$

$$P_{y_2} = P_{y_1} \quad (VII)$$

or, since

$$\alpha_k = \alpha_{E_2} - \alpha_{E_1}$$

$$P_{x_2} = P_{x_1} \cos (\alpha_{E_2} - \alpha_{E_1}) - P_{z_1} \sin (\alpha_{E_2} - \alpha_{E_1}) \quad (VIII)$$

$$P_{z_2} = P_{z_1} \cos (\alpha_{E_2} - \alpha_{E_1}) + P_{x_1} \sin (\alpha_{E_2} - \alpha_{E_1}) \quad (IX)$$

$$P_{y_2} = P_{y_1} \quad (X)$$

For flight-mechanic applications, this simple transfer from one system of body axes to another, has a substantial advantage.

The problem involved is as follows: The forces and moments of an airfoil or of a complete airplane model, are recorded by wind-tunnel tests. The recorded coefficients are then to be introduced in the motion and stability equations. While the coefficients in the tunnel tests referred to any one system of axes deemed suitable in practical application, the moment equations are usually referred to the principal axes of inertia of the airplane. To refer these coefficients to principal axes of inertia in tunnel tests, is usually unsuccessful for the reason that in a tunnel study the actual position of the principal axes of inertia of the subsequent full-scale design of the airplane, is not known at all.

The problem therefore consists in transferring the coefficients from one system - say,  $e$  axes - to another system of body axes. Likewise, a change in the position of the principal axes of inertia of the same airplane, for instance, by subsequent structural changes, may make it necessary to transfer the coefficients or angular velocities referred to principal axes of inertia  $x_1, y, z_1$  to another system  $x_2, y, z_2$ , whose  $x_2, z_2$  axes are rotat-

ed through angle  $\alpha_K$  in the plane of symmetry with respect to  $x_1, z_1$ .

This shows that the use of the coefficients from wind-tunnel tests in flight-mechanic equations is particularly simple with the aid of the angles of the E coupling; hence, from this point of view the choice of the E coupling in the mathematical treatment of flight-mechanic problems, is advantageous. Another advantage accrues from the fact that, because of the simple transfer from one system of body axes to another, no previous body axes  $x, z$  are necessary. This latter advantage became particularly apparent in a generalized representation of the lateral stability theory where, if the equations are written so that the derivatives of moments  $L$  and  $N$  are referred to  $e$  axes, the representation can be largely formulated independent of the size of the angle of attack. It merely involves, then, a simple transformation of the derivatives of  $e$  axes to body axes; the transfer from axes  $x_0, z_0$  to the principal axes of inertia  $x, z$  is extremely simple with the aid of angle  $\alpha_E$  between  $x_0$  and  $x$ . By effecting this transformation, the size of  $\alpha_E$  is, of course, unrestricted. This is a substantial advantage, especially when formulating the lateral stability equation, where it becomes evident that the effectuated omissions in nowise depend upon the size of the angle of attack.

The F coupling between body and wind axes offers far less advantages. The angles by which a model, and hence its related axes,  $x, y, z$ , is directed with respect to the tunnel axes, are the angles  $\alpha_E$  and  $\beta_E$  of the E coupling. If, in the numerical treatment of flight-mechanic equations the angles  $\alpha_F$  and  $\beta_F$  of the F coupling are employed, the transfer of the tunnel data to flight-mechanic applications requires a conversion of the quantities recorded in the tunnel which, as will be shown, is quite complicated.

From the equations (I) to (XII), the following equations between the angles of the F and E couplings can be derived:

$$\sin \alpha_F = \sin \alpha_E \cos \beta_E \quad (XI)$$

$$\sin \beta_E = \sin \beta_F \cos \alpha_F \quad (XII)$$

$$\tan \beta_F = \frac{\tan \beta_E}{\cos \alpha_E} \quad (\text{XIII})$$

$$\tan \alpha_E = \frac{\tan \alpha_F}{\cos \beta_F} \quad (\text{XIV})$$

From (XI) and (XIV) or equations (V) and (VIII), follows:

$$\cos \alpha_E \cos \beta_E = \cos \alpha_F \cos \beta_F \quad (\text{XV})$$

In addition, it requires a table for transferring directed quantities from the wind axes  $x_a, y_{aE}, z_{aE}$  defined by the E coupling, to the wind axes  $x_a, y_{aF}, z_{aF}$  defined by the F coupling.

The table is obtained by the following method: Visualize figures 4 and 6 plotted together, while bearing in mind that the four axes  $y_{aE}, y_{aF}, z_{aE}, z_{aF}$  are situated in the same plane, namely, the cross-wind plane at right angles to  $x_a$ . In this instance, the following relations hold:

$$y_{aE} = y_{aF} \cos \delta + z_{aF} \sin \delta \quad (1)$$

$$z_{aE} = z_{aF} \cos \delta - y_{aF} \sin \delta \quad (2)$$

$\delta$  denotes the angle between  $z_{aE}$  and  $z_{aF}$ , and  $y_{aE}$  and  $y_{aF}$ , respectively.

Since the plane  $z_{aE}, z_{aF}$  is at right angles to plane  $x, x_a$  in which  $\alpha_F$  is measured, it follows for  $\cos \delta$ :

$$\cos \delta = \frac{\cos \alpha_E}{\cos \alpha_F}$$

and, since planes  $xy$  and  $x_a y_{aE}$  meet at angle  $\alpha_E$ , it follows from triangle  $y y_{aF} y_{aE}$ :

$$\sin \delta = \sin \alpha_E \sin \beta_F$$

Thus the complete transfer from  $x_a, y_{a_F}, z_{a_F}$  to  $x_a, y_{a_E}, z_{a_E}$  has the form given in

TABLE VIII

Wind axes E coupling	Wind Axes (F Coupling)		
	$x_a$	$y_{a_F}$	$z_{a_F}$
	$x_a$	1	0
	$y_{a_E}$	0	$\frac{\cos \alpha_E}{\cos \alpha_F}$
	$z_{a_E}$	0	$-\sin \alpha_E \sin \beta_F$
			$\frac{\cos \alpha_E}{\cos \alpha_F}$

From (XI) to (XV) further, follow:

$$\frac{\cos \alpha_E}{\cos \alpha_F} = \frac{\cos \beta_F}{\sqrt{1 - \cos^2 \alpha_F \sin^2 \alpha_E}} \quad (\text{XVI})$$

$$\sin \alpha_E \sin \beta_E = \frac{\sin \alpha_F \sin \beta_F}{\sqrt{1 - \cos^2 \alpha_F \sin^2 \beta_F}} \quad (\text{XVII})$$

$$\frac{\cos \alpha_E}{\cos \alpha_F} = \frac{\cos \alpha_E}{\sqrt{1 - \sin^2 \alpha_E \cos^2 \beta_E}} \quad (\text{XVIII})$$

$$\sin \alpha_E \sin \beta_F = \frac{\sin \alpha_E \sin \beta_E}{\sqrt{1 - \sin^2 \alpha_E \cos^2 \beta_E}} \quad (\text{XIX})$$

Through these formulas table VIII can equally well be written with the angles of the E coupling or those of the F coupling.

Now the following case is analyzed: The coefficients  $c_{x_e}, c_{y_e}, c_{z_e}$  recorded in the wind tunnel are, as usual, referred to e axes. The equations are to be computed

with the angles of the F coupling  $\alpha_F$  and  $\beta_F$  and with principal inertia axes. The recorded wind-tunnel coefficients must therefore be transferred to principal inertia axes  $x, y, z$ .

Based upon the angles  $\alpha_E$  and  $\beta_E$ , the conversion formulas for the coefficients  $c_x, c_y, c_z$  referred to the principal inertia axes, read as follows:

$$c_x = c_{x_e} \cos \alpha_E - c_{z_e} \sin \alpha_E \quad (XX)$$

$$c_y = c_{y_e} \quad (XXI)$$

$$c_z = c_{z_e} \cos \alpha_E + c_{x_e} \sin \alpha_E \quad (XXII)$$

Then with  $\alpha_F$  and  $\beta_F$  from equations (XVI) and (XVII) instead of  $\alpha_E$ , the three conversion formulas read:

$$c_x = c_{x_e} \frac{\cos \alpha_F \cos \beta_F}{\sqrt{1 - \cos^2 \alpha_F \sin^2 \beta_F}} - c_{z_e} \frac{\sin \alpha_F}{\sqrt{1 - \cos^2 \alpha_F \sin^2 \beta_F}} \quad (XXa)$$

$$c_y = c_{y_e} \quad (XXIa)$$

$$c_z = c_{z_e} \frac{\cos \alpha_F \cos \beta_F}{\sqrt{1 - \cos^2 \alpha_F \sin^2 \beta_F}} + c_{x_e} \frac{\sin \alpha_F}{\sqrt{1 - \cos^2 \alpha_F \sin^2 \beta_F}} \quad (XXIIa)$$

It is readily apparent that the use of the F coupling entails a lot of paper work; and the superiority of the E coupling with respect to simplicity of conversion when transferring from one system of body axes  $x_1, y_1, z_1$  to another  $x_2, y_2, z_2$ , is also plain. Clearly, such a transfer is much more readily accomplished with the E coupling, since a rotation of axes  $x, z$  in the plane of symmetry of the airplane, modifies only one of the two angles ( $\alpha_E$ ); while with the F coupling, both  $\alpha_F$  and  $\beta_F$  are changed. This is due to the fact that in the case of the E coupling, the plane fixed to the body, as used for defining the aerodynamic angles, is the plane of symmetry  $zx$ , which does not change position by a rotation of the axes  $z, x$  but changes if the F coupling is resorted to.



The transfer of directed quantities from one system of body axes to another by the F-coupling method, therefore, is accompanied by the appearance of both  $\alpha_F$  and  $\beta_F$  in the conversion formulas which are, in consequence, quite complicated and difficult to present.

For the derivation of these reduction formulas, the corresponding equations for the E coupling, applicable when transferring from  $x_1, z_1$  to  $x_2, z_2$ , are repeated:

$$x_2 = x_1 \cos (\alpha_{E_2} - \alpha_{E_1}) - z_1 \sin (\alpha_{E_2} - \alpha_{E_1}) \quad (XXIII)$$

$$z_2 = z_1 \cos (\alpha_{E_2} - \alpha_{E_1}) + x_1 \sin (\alpha_{E_2} - \alpha_{E_1}) \quad (XXIV)$$

or

$$\begin{aligned} x_2 = & x_1 (\cos \alpha_{E_2} \cos \alpha_{E_1} + \sin \alpha_{E_2} \sin \alpha_{E_1}) \\ & - z_1 (\sin \alpha_{E_2} \cos \alpha_{E_1} - \cos \alpha_{E_2} \sin \alpha_{E_1}) \end{aligned} \quad (XXIIIa)$$

$$\begin{aligned} z_2 = & z_1 (\cos \alpha_{E_2} \cos \alpha_{E_1} + \sin \alpha_{E_2} \sin \alpha_{E_1}) \\ & + x_1 (\sin \alpha_{E_2} \cos \alpha_{E_1} - \cos \alpha_{E_2} \sin \alpha_{E_1}) \end{aligned} \quad (XXIVa)$$

The transfer from  $\alpha_{E_1}, \alpha_{E_2}$  to  $\alpha_{F_1}, \alpha_{F_2}$ , is achieved by means of formulas (XVI) and (XVII). It gives

$$\cos \alpha_{E_1} = \frac{\cos \alpha_{F_1} \cos \beta_{F_1}}{\sqrt{1 - \cos^2 \alpha_{F_1} \sin^2 \beta_{F_1}}} \quad (XVIIa)$$

$$\sin \alpha_{E_1} = \frac{\sin \alpha_{F_1}}{\sqrt{1 - \cos^2 \alpha_{F_1} \sin^2 \beta_{F_1}}} \quad (XVIIa)$$

Corresponding formulas for  $\alpha_{E_2}$  can be written.

The expressions for  $\sin \alpha_{E_1}, \cos \alpha_{E_1}, \sin \alpha_{E_2}, \cos \alpha_{E_2}$  (equations (XXIII) and (XXIV)) written in, give:

$$\begin{aligned}
 x_2 = x_1 & \frac{\cos \alpha_{F_1} \cos \alpha_{F_2} \cos \beta_{F_1} \cos \beta_{F_2} + \sin \alpha_{F_1} \sin \alpha_{F_2}}{\sqrt{(1 - \cos^2 \alpha_{F_1} \sin^2 \beta_{F_1})(1 - \cos^2 \alpha_{F_2} \sin^2 \beta_{F_2})}} \\
 - z_1 & \frac{\cos \alpha_{F_1} \cos \beta_{F_1} \sin \alpha_{F_2} - \sin \alpha_{F_1} \cos \alpha_{F_2} \cos \beta_{F_2}}{\sqrt{(1 - \cos^2 \alpha_{F_1} \sin^2 \beta_{F_1})(1 - \cos^2 \alpha_{F_2} \sin^2 \beta_{F_2})}} \quad (XXV)
 \end{aligned}$$

$$\begin{aligned}
 z_2 = z_1 & \frac{\cos \alpha_{F_1} \cos \alpha_{F_2} \cos \beta_{F_1} \cos \beta_{F_2} + \sin \alpha_{F_1} \sin \alpha_{F_2}}{\sqrt{(1 - \cos^2 \alpha_{F_1} \sin^2 \beta_{F_1})(1 - \cos^2 \alpha_{F_2} \sin^2 \beta_{F_2})}} \\
 + x_1 & \frac{\cos \alpha_{F_1} \cos \beta_{F_1} \sin \alpha_{F_2} - \sin \alpha_{F_1} \cos \alpha_{F_2} \cos \beta_{F_2}}{\sqrt{(1 - \cos^2 \alpha_{F_1} \sin^2 \beta_{F_1})(1 - \cos^2 \alpha_{F_2} \sin^2 \beta_{F_2})}} \quad (XXVI)
 \end{aligned}$$

According to these formulas, the use of the E coupling affords substantial advantages by enabling a convenient transfer from one system of body axes to another, with the aid of a single angle measured in the plane of symmetry.

Hopf's (reference 2) claim of superiority of the F coupling over the E coupling is, that the resultant air load - based upon the F coupling - is largely dependent upon the angle of attack only, and is not changed by small angles of yaw. Citing a circular plate as example, he states that in a rotation of the disk about the axis, erected in the disk center at right angles to the disk surface, the position of the disk in regard to the flow and hence on the flow conditions, remains the same. This objection to the E coupling is now investigated from the point of view of practical application.

Since on an airplane it does not entail a rotationally symmetrical body, as on the cited circular disk, it may be expected that the independence of the resultant aerodynamic force - that is, principally of the lift from an angle of yaw  $\beta_F$  - really exists only within a very restricted range of  $\beta_F$ . Naturally, there always will be an axis (optimum axis) for a certain airfoil in the angle-of-attack

range of normal, unstalled flight which has the property that, in a rotation of the wing about this axis the lift coefficient  $c_a$  has a minimum of dependence upon this rotation. Complete independence of lift upon angle of yaw in unsymmetrical flow is not present, even in a small range of  $\beta_F$ , as long as the  $c_a$  values are no longer small - this, even if the cited optimum axis is chosen as axis of rotation for  $\beta_F$ . For an untwisted wing of the usual contours, this optimum axis is in approximate agreement with the axis at right angles to the plane of the chord which passes through the center of gravity.

It will be shown on three examples from Göttingen wind-tunnel tests, to what extent the lift depends on the angle of yaw if one or the other angle coupling is used as basis.

The first measurement was made on the complete model of a high-wing landplane. The  $c_a$ ,  $c_q$ ,  $c_w$  recorded at different  $\alpha$  in relation to the angle of yaw, are used. The angle of yaw was varied from  $0^\circ$  to  $45^\circ$ .

The second measurement was made on the complete model of a high-wing flying boat in the same way as on the landplane model; the angle of yaw varied from  $0^\circ$  to  $15^\circ$ .

The third measurement was made in the same way on the model of a low-wing landplane. The angles used in the Göttingen measurements to orientate the model with respect to the flow, are  $\alpha_E$  and  $\beta_E$ . The recorded lift coefficient is  $c_{aE}$ .

The transfer from  $c_{aE}$  to  $c_{aF}$  was effected by transformation of angles  $\alpha_E$  and  $\beta_E$  describing a certain state of flow into  $\alpha_F$  and  $\beta_F$  with the help of formulas (XI) to (XV). Then  $c_{aF}$  is computed from  $c_{aE}$  and  $c_{qE}$  by means of table VIII. By this complete transformation of the angles and forces, the pertinent flow condition can then be described by the angles of the F coupling and related wind axes  $x_a$ ,  $y_{aF}$ ,  $z_{aF}$  defined by the F coupling.

In this manner a clear picture may be gained of the extent of dependence existing in a number of flow conditions, between  $c_{aE} = f(\beta_E)$  and  $c_{aF} = f(\beta_F)$ . But even then, no direct comparison of the curves  $c_{aE} = f(\beta_E)$  and

$c_{aF} = f(\beta_F)$  can be effected. While  $\alpha_E$  is constant during the measurement and  $\beta_E$  varies, it follows from the conversion that  $\beta_F$ , as well as  $\alpha_F$  is variable, so that a  $c_{aF}^* = f(\beta_F)$  obtained from the conversion must, because of the variable  $\alpha_F$ , be corrected.  $c_{aF}$  becomes smaller, not only as a result of the rising angle of yaw but also owing to the decrease in angle  $\alpha_F$ , caused by the conversion. The correction for  $c_{aF}$  was made with the aid of the curve  $c_{aE} = f(\alpha_E)$  for  $\beta_E = 0$  since, in the case of symmetrical flow - i.e., absence of angle of yaw -  $c_{aE} = c_{aF}$ .

Figure 8 shows the curve  $c_{aE} = f(\beta_E)$  for a high-wing landplane as recorded in the wind tunnel, and the corrected  $c_{aF} = f(\beta_F)$ . The body reference axis for  $\alpha_E$  is the chord axis.

The curves disclose the following:

1. Complete independence of  $c_{aF}$  from  $\beta_F$  does not exist, even in the region of small angles of yaw.
2. The curves  $c_{aE} = f(\beta_E)$  and  $c_{aF} = f(\beta_F)$  are not materially different in the range of small angles of yaw. At  $11.5^\circ$  angle of attack and  $20^\circ$  angle of yaw,  $c_{aE}$  has dropped to 88.2 percent of the value recorded at  $\beta = 0$ , and  $c_{aF}$  to 91.5 percent of the value recorded at  $\beta = 0$ .
3. While  $c_{aE}$  drops much more than  $c_{aF}$  at higher angles of yaw, the absolute drop of both  $c_{aE}$  and  $c_{aF}$  is substantial. Accordingly, the choice of  $c_{aF}$  by  $\beta_F > 20^\circ$  presents no material advantage over  $c_{aE}$ .

It should also be remembered that the body reference axis of  $\alpha_E$  was the chord axis. Hence, a conversion to the angles of the F coupling gives an angle  $\beta_F$  which has a body normal axis perpendicular to the plane of the chord

as axis of rotation. Herewith the measurements are referred to the previously quoted approximate optimum axis, and the presented dependence  $c_{aF} = f(\beta_F)$  approximately represents the most favorable case.

Figures 9 and 10 show measurements on model high-wing seaplanes and low-wing landplanes, taken at different angles of attack. The curves of  $c_{aE} = f(\beta_E)$  and  $c_{aF} = f(\beta_F)$  are fundamentally similar for the most dissimilar types. The relation of lift to angle of yaw within 0 to 20° angle of yaw is similar in order of magnitude. One generalization at least is readily apparent, namely, that even with the F coupling as choice, a complete independence of lift from angle of yaw does not exist even at small angles of yaw, up to about 20° if no small  $c_a$  value is present. In this case, however,  $c_{aE}$  is practically as little variable as  $c_{aF}$  in the range of small  $\beta$ , and then the F coupling presents no appreciable superiority over the E coupling.

In this connection, the only case encountered in practice where large  $\beta$  occurs needs mentioning, namely, the case of the seaplane afloat on water and subject to any angle of yaw.

In this range of large  $\beta$ , two objections may be raised against the use of the E coupling:

1. The position of lift axis  $z_{aE}$  is no longer definitely defined at  $\beta_E = 90^\circ$ ;
2. The lift  $c_{aE}$  is markedly affected by  $\beta_E$  and becomes, in fact, altogether independent at  $\beta_E = 90^\circ$ , because in this case the axis of rotation of angle  $\alpha_E$  (y axis) coincides with the wind axis. Anent these two objections, the following should be noted:

1. The position of lift axis  $z_{aE}$  becomes undetermined only when  $\beta_E$  actually amounts to exactly  $90^\circ$ . At the least departure from it,  $z_{aE}$  is definitely fixed.  $z_{aE}$  is wholly independent of the magnitude of the angle of yaw in accord with  $z_e$  - i.e., with the axis of rotation of angle  $\beta_E$ , whose position is practically always defined.

Moreover, with the F coupling for axis  $y_{aF}$ , the conditions are entirely similar when  $\sigma_F = -90^\circ$ ; and  $\alpha_F = 90^\circ$  is attainable in a flat spin. So from this point of view, the F coupling is nowise more propitious than the E coupling.

2. According to figures 8, 9, and 10, a passable independence of  $c_{aF}$ , as of  $c_{aE}$  from the angle of yaw prevails only at very small angles of yaw. Hence, in the range of high angles of yaw, the dependence of the lift upon the angle of yaw must be experimentally ascertained for the different values of angle of attack, no matter what coupling is chosen between the body and the wind axes.

However, the two objections cited against the E coupling at large angles of yaw can be ignored from the very beginning for a much deeper reason; for the large angles of yaw are practically confined to seaplanes floating on the water. And in this case, stability problems of the seaplane as floating body are usually involved rather than a purely flight-mechanic application. Then the seaplane is, in fact, orientated in a plane which in the practical application, is given by the surface of the water, in the wind-tunnel test by a ground plate. This implies, however, that the body axes in this instance must be orientated with respect to a second complete system of axes, two axes of which are situated in the plane of the water surface or ground plate, respectively, while the third axis is at right angles to the other two. This orientation requires, as described in reference 9, three Euler angles which generally are other than zero, whereas in the case of orientation of the body axes relative to a system of wind axes - of which, for the time being, the position of only one axis (wind axis) is given - one of the three Euler angles, through a certain arbitrary tie of wind axis  $y_a$  or  $z_a$  to the body axis, is zero. This special case of orientation by two angles only is utterly inapplicable to the complete orientation of an airplane relative to a ground plate. For by the orientation of one system of axes with respect to another by means of three Euler angles, it is impossible to prescribe beforehand a certain tie of an axis of the orientating system to a plane of the axes system to be orientated - as achieved in the case of coupling between body axes and wind axes.

The orientation of a seaplane afloat on the water requires the use of an angle that enables, at  $\beta = 90^\circ$ , the

definition of the inclination of the wings with respect to the surface of the water. This case therefore calls for an extension of the coupling between the body and wind axes. The appropriate coupling angle to be used in this case is described in section IV.

#### IV. EXTENSION OF THE COUPLING BETWEEN THE BODY AND WIND AXES TO INCLUDE THE CASE OF AN AIRPLANE IN A FLOW OVER A GROUND PLATE

The case of an airplane in a flow above a ground plate - while being orientated in respect to the flow and the ground plate - is practically realized by a seaplane afloat on the water. (Since this case involves aerodynamic problems of static stability, the presence of a ground plate - i.e., surface of water in the case of a seaplane - is physically conditioned and the orientation must be made with regard to this surface.)

This case goes beyond the previously described coupling between a system of body axes and a system of wind axes achieved by two angles. Through the given surface, a complete system of axes is given as orientating system, in respect to which the body axes are directed by three Euler angles. The orientating axes ( $x_{ap}$ ,  $y_{ap}$ ,  $z_{ap}$ ) conditioned by the ground plate, are explained as follows:

The origin of axes  $x_{ap}$ ,  $y_{ap}$ ,  $z_{ap}$  is placed in the center of gravity of the airplane; that is, the ground plate is, similar to the ground axes, shifted parallel along the earth's vertical axis into the center of gravity of the airplane.

Axis  $x_{ap}$  agrees with the axis of flow located in the plane of the plate (plane of water surface).

Axis  $y_{ap}$  is at right angles to  $x_{ap}$  in the plane of the plate.

Axis  $z_{ap}$  is at right angles to  $x_{ap}$  and  $y_{ap}$ ; i.e., also at right angles to the plane of the plate.

The axis of flow, being always in the plane of the plate,  $x_{ap}$  itself is at right angles to the axis of flow.

An examination of wind-tunnel data with and without ground plate, discloses the following:

Generally, the plane of plate  $x_{ap} y_{ap}$  agrees with the resistance transverse force plane  $x_{aK} y_{aK}$  of the tunnel, because the axis of rotation of angle  $\beta_E(z_{aK})$  is at right angles to the plane of the plate. In practice, the choice of the E coupling, therefore, in which the wind axes of the tunnel  $x_{aK}, y_{aK}, z_{aK}$  are identical with the wind axes  $x_a, y_a, z_a$  used in free flight, would make the surface of the water correspond with the tangential flow plane  $x_a y_{aK}$ ; that is,  $z_{aE}$  would agree with  $z_{ap}$ . In reality, however, the conditions between the plane of the plate  $x_{ap} y_{ap}$  and the plane  $x_{aK} y_{aK}$  of the drag transverse force, manifest a fundamental difference despite formal agreement. The position of axes  $y_{aK}$  and  $z_{aK}$  and hence, that of plane  $x_{aK} y_{aK}$  (or  $x_a y_{aE}$ ) is at first physically unimportant, and is only defined in the above-described manner by one of the two couplings for reasons of expediency. Only the flow axis  $x_{aK}$  is of practical significance in wind-tunnel tests without ground plate, which corresponds to the conditions of an airplane in free flight. To direct this axis to the body system requires only two angles. On the other hand, the ground plate  $x_{ap} y_{ap}$  (in practical application to seaplanes, the surface of the water) represents, through the special type of problems involved in this case, a physical opportunity which is definitely known from the very start. To direct the body axes to this plane of the plate, requires however, three angles.

Through the formal agreement between the plane of the plate and  $x_{aK} y_{aK}$ , the orientating axes given by the ground plate, can be explained as a special type of wind axes of the tunnel. For  $x_{ap} = x_{aK}, z_{ap} = z_{aK}$ , hence  $y_{ap} = y_{aK}$ . By proper transfer of the E coupling to the discussed orientation of the body axes to the orientating



axes  $x_{ap}$ ,  $y_{ap}$ ,  $z_{ap}$  given by the flow, the principal axis of rotation of the orientating system is axis  $z_{ap} = z_{ax}$ , and the definitive axis of rotation of the body system to be orientated is formed by axis  $y$ . With the axes of rotation for two of the three coupling angles known, it becomes readily apparent that the angles  $\alpha_E$  (axis of rotation  $y$ ) and  $\beta_E$  (axis of rotation  $z_{ax} = z_{ax}$ ) in their original definition, also appear as coupling angles between the body axes and the axes of the ground plate. The third angle is  $\varphi_E$ , termed "angle of roll of the tunnel," whose axis of rotation is immediately given by the junction line between body axes  $x$ ,  $y$ ,  $z$  and the ground-plate axes  $x_{ap}$ ,  $y_{ap}$ ,  $z_{ap}$ . This junction line is in formal agreement with axis  $x_E$  of the axes of the plane of symmetry.

Now it is to be noted that axis  $z_{ax}$ , in whose direction the lift is measured is, in the presence of an angle  $\varphi_E$ , no longer in the plane of symmetry of the airplane, but rather that the plane of symmetry inclines at angle  $\varphi_E$  with respect to axis  $z_{ap} = z_{ap}$ . The premise that the lift axis shall be at right angles to the axis of flow in the plane of symmetry of the airplane can, in this case, no longer be positively maintained since, by orientation of one system of axes with respect to another complete system by three Euler angles, every prescribed anchorage of one axis of the orientating system to a plane of the system to be orientated represents an agreement and, consequently, a contradiction. Such a prescribed anchorage is timely and necessary only in the case where a system is to be orientated with respect to a known axis.

The application of the conditions obtaining in wind-tunnel tests on a model with ground plate, to the practical case of a seaplane afloat on the water can be effected forthwith if, as in the wind tunnel, angles  $\alpha_E$  and  $\beta_E$  are identical with two of the three coupling angles between the body axes and the axes of the water surface. In this case, the axes  $x_{ap}$ ,  $y_{ap}$ ,  $z_{ap}$  of the system describing the flow on the seaplane, became the axes  $x_{ax}$ ,  $y_{ax}$ ,  $z_{ax}$  of the tunnel system.  $x_{ap}$  and  $y_{ap}$  are in the plane of the water surface or parallel to it. The axes

system  $x_{ap}, y_{ap}, z_{ap}$  is, as in the wind tunnel, termed the "wind axes system for measurements with ground plate."

The seaplane is orientated with respect to the water surface through the following three angles:

- $\alpha_E$  angle of attack: The angle between body x axis and the line of intersection between airplane plane of symmetry and plane  $x_{ap} y_{ap}$  (plane parallel to water surface through the center of gravity). The body y axis is the axis of rotation of the angle, the plane of symmetry of the airplane, the plane of measurement.
- $\beta_E$  angle of yaw: The angle between axis  $x_{ap}$  and the line of intersection of plane  $x_{ap} y_{ap}$  with the plane of symmetry of the airplane. Axis of rotation of the angle is axis  $z_{ap}$  at right angles to water surface; plane of measurement the water surface or one parallel to it.
- $\varphi_E$  angle of roll: Angle between axis  $z_{ap}$  and the plane of symmetry of the airplane. Axis of rotation of angle is the trace between plane of symmetry of airplane and plane  $x_{ap} y_{ap}$ . Plane of measurement is plane  $y z_{ap}$ .

The line of intersection between the plane of symmetry of the airplane and plane of the plate  $x_{ap} y_{ap}$  occurring by the definition of  $\alpha_E$  and  $\beta_E$  and forming the axis of rotation for  $\varphi_E$  is none other than axis  $x_E$  of the experimental system.

The relations between the direction cosine and the coupling angles  $\alpha_E, \beta_E, \varphi_E$  between the body axes  $x, y, z$  and the wind axes  $x_{ap}, y_{ap}, z_{ap}$  in measurements with a ground plate, can be deduced from figure 11 with the aid of the cosine law. All angles shown are positive.

The following relations hold true:

From triangle  $xF x_{ap}$ : 1.  $\cos(x x_{ap}) = \cos \alpha_E \cos \beta_E$   
 $-\sin \alpha_E \sin \beta_E \sin \varphi_o$

From triangle  $xF y_{ap}$ : 2.  $\cos(x y_{ap}) = \cos \alpha_E \sin \beta_E$   
 $+\cos \beta_E \sin \alpha_E \sin \varphi_o$

From triangle  $xB z_{ap}$ : 3.  $\cos(x z_{ap}) = -\sin \alpha_E \cos \varphi_o$

From triangle  $yA x_{ap}$ : 4.  $\cos(y x_{ap}) = -\sin \beta_E \cos \varphi_o$

From triangle  $yA y_{ap}$ : 5.  $\cos(y y_{ap}) = \cos \beta_E \cos \varphi_o$

6.  $\cos(y z_{ap}) = \sin \varphi_o$


From triangle  $zF x_{ap}$ : 7.  $\cos(z x_{ap}) = \sin \alpha_E \cos \beta_E$   
 $+\cos \alpha_E \sin \beta_E \sin \varphi_o$

From triangle  $zF y_{ap}$ : 8.  $\cos(z y_{ap}) = \sin \alpha_E \sin \beta_E$   
 $-\cos \alpha_E \cos \beta_E \sin \varphi_o$

From triangle  $zB z_{ap}$ : 9.  $\cos(z z_{ap}) = \cos \alpha_E \cos \varphi_o$

The transfer from body axes to wind axes in presence of a ground plate, hence of an angle of roll  $\varphi_o$ , has the form shown in

TABLE IX

		Wind Axes for Measurements with Ground Plate		
		$x_{ap}$	$y_{ap}$	$z_{ap}$
Body axes	x	$\cos \alpha_E \cos \beta_E$ $-\sin \alpha_E \sin \beta_E \sin \varphi_o$	$\sin \beta_E \cos \alpha_E$ $+\cos \beta_E \sin \alpha_E \sin \varphi_o$	$-\sin \alpha_E \cos \varphi_o$
	y	$-\sin \beta_E \cos \varphi_o$	$\cos \beta_E \cos \varphi_o$	$\sin \varphi_o$
	z	$\sin \alpha_E \cos \beta_E$ $+\cos \alpha_E \sin \beta_E \sin \varphi_o$	$\sin \alpha_E \sin \beta_E$ $-\cos \alpha_E \cos \beta_E \sin \varphi_o$	$\cos \alpha_E \cos \varphi_o$

Equating  $\phi_0$  to 0, table IX becomes table V, which contains the transfer from body axes  $x y z$  to wind axes  $x_a y_a z_a$  used with the airplane in free flight.

But there is yet another coupling possibility, for on selecting axis  $x$  of the aircraft system as definitive axis of rotation, the orientation of the aircraft system with respect to axes  $x_g, y_g, z_g$  given by the ground plate and the flow, is made through three angles, which formally agree with the angles  $\phi, \theta, \psi$  (explained in (VI)), by which the body axes are directed with respect to the ground axes. By this coupling the ground plate is supposed to explain a system of ground axes (reference 10); thereby axis  $x_g$  is situated in the horizontal plane passing through the center of gravity and is in agreement with the axis of the wind stream. The drawback of this coupling is that axis  $x$  as definitive axis of rotation, whose position is never definitely known from the start, is arbitrarily defined from one case to the next.

The third coupling possibility, in which axis  $z$  of the body axes system is chosen as definitive axis of rotation, proves impractical. The related Cardan model would in neutral setting (all three coupling angles = 0) disclose parallel rings.

# V. THE COUPLING BETWEEN RELATIVE WIND AXES AND BASIC GROUND SYSTEM, AND DISCUSSION OF THE COUPLING OF A SYSTEM FIXED TO THE FLIGHT PATH WITH THE BASIC GROUND SYSTEM

By the coupling of the wind axes  $x_a, y_a, z_a$  with the ground axes  $x_g, y_g, z_g$ , the ground axes shall become, in conformity with the earlier statements - regarding the orientation of the individual systems - the orientating axes for the wind axes to be orientated. The one axis of the ground system with definitely known position from the start, is the vertical axis  $z_g$ . In consequence,  $z_g$  represents the natural principal axis of rotation of the orientating system  $x_g, y_g, z_g$ .

Selection of the definitive axis of rotation is not difficult with the wind system. Physically conditioned and known in position, for the present, is the relative wind axis  $x_a$  only. The position of  $y_a$  and  $z_a$  is, at first arbitrarily established, whereby the individual couplings afford various possibilities. By choosing axis  $x_a$  as definitive axis of rotation, the three coupling angles between the wind system and the ground system are explained as follows:

$\gamma$  relative wind longitudinal angle of inclination - the angle between  $x_a$  and plane  $x_g y_g$ : Axis of rotation of angle is axis  $y_a$  projected on horizontal plane  $x_g y_g$ ; i.e., a horizontal axis at right angles to  $x_a$ , whose positive sense of direction viewed against the flow, is to the right. Plane  $x_a z_g$  is the plane of measurement.

$\mu$  relative wind lateral angle of inclination - angle between  $z_a$  and plane  $x_a z_g$ : Axis  $x_a$  is axis of rotation of the angle. Cross-wind plane  $y_a z_a$  is plane of measurement.

$\chi$  relative wind azimuth angle - angle between  $x_a$  projected on plane  $x_g y_g$  and axis  $x_g$ : Axis  $z_g$  is axis of rotation of the angle. Plane  $x_g y_g$  is the plane of measurement.

The application of the cosine law gives, as seen from figure 12, the following relations between the direction cosine and the coupling angles  $\gamma$ ,  $\mu$ ,  $\chi$ .

From triangle  $x_a F x_g$ : 1.  $\cos(x_a x_g) = \cos \gamma \cos \chi$

From triangle  $x_a F y_g$ : 2.  $\cos(x_a y_g) = \cos \gamma \sin \chi$   
3.  $\cos(x_a z_g) = -\sin \gamma$

From triangle  $y_a K x_g$ : 4.  $\cos(y_a x_g) = -\sin \chi \cos \mu$   
 $+ \cos \chi \sin \mu \sin \gamma$

From triangle  $y_a K y_g$ : 5.  $\cos(y_a y_g) = \cos \chi \cos \mu$   
 $+ \sin \chi \sin \mu \sin \gamma$

From triangle  $y_a x_a z_g$ : 6.  $\cos(y_a z_g) = \cos \gamma \sin \mu$

From triangle  $z_a$  K  $x_g$ : 7.  $\cos(z_a x_g) = \sin \mu \sin \chi$   
 $+ \cos \mu \cos \chi \sin \gamma$

From triangle  $z_a$  K  $y_g$ : 8.  $\cos(z_a y_g) = -\sin \mu \cos \chi$   
 $+ \cos \mu \sin \chi \sin \gamma$

From triangle  $z_a$   $x_a$   $z_g$ : 9.  $\cos(z_a z_g) = \cos \gamma \cos \mu$

Now the transfer from wind axes  $x_a, y_a, z_a$  to ground axes  $x_g, y_g, z_g$  has the form of

TABLE I

		G r o u n d   A x e s		
		$x_g$	$y_g$	$z_g$
Wind axes	$x_a$	$+\cos \gamma \cos \chi$	$+\cos \gamma \sin \chi$	$-\sin \gamma$
	$y_a$	$+\sin \gamma \cos \sin \mu$ $-\sin \chi \cos \mu$	$+\sin \gamma \sin \chi \sin \mu$ $+\cos \chi \cos \mu$	$+\cos \gamma \sin \mu$
	$z_a$	$+\sin \gamma \cos \chi \cos \mu$ $+\sin \chi \sin \mu$	$+\sin \gamma \sin \chi \cos \mu$ $-\cos \chi \sin \mu$	$+\cos \gamma \cos \mu$

Wind changes and upwind disregarded, the wind axis  $x_a$  agrees with the tangent to the flight path. In this case  $z_a$  is defined as the path normal situated in the plane of symmetry of the airplane. If wind changes are considered, the angle of inclination of the flight path tangent with respect to the ground as well as the angle formed by its projection on the ground with a certain initial direction, is no longer in agreement with  $\gamma$  and  $\chi$ . In this instance  $\gamma$  and  $\chi$  must carry a subscript (for instance,  $B = \text{path}$ ). To describe flight-path motions which are to be achieved without consideration of the force coefficients  $c_w, c_q, c_a$ , a special system of path axes can also be defined, which is then orientated with respect to the ground. Such a system has been proposed by the International Commission for Air Navigation (CINA) as a natural system of path axes. The axes of this system are the tangent, the principal normal, and the binormal of the three-dimensional flight path. The principal normal is the normal situated in the plane formed by two infinitesimally adjacent tangents.

The orientation of these axes with respect to those parallel to the ground system, is the same as that of the wind system  $x_a, y_a, z_a$ .

In all cases where the force coefficients  $c_w, c_q, c_a$  are to be introduced, the wind system is preferable and orientated with respect to the basic ground system, the wind changes usually being ignored and axis  $x_a$  therefore, assumed to be in agreement with the path tangent. The use of the natural axes of the flight path has, undoubtedly, certain advantages in calculations dealing principally with the flight path. But for the representation of the force equations with  $c_w, c_q, c_a$ , it is ill-suited since it requires another reference angle for orientating the system  $c_q, c_a$  with respect to the principal normal plane.

In conclusion, it is pointed out that the magnitude of angle  $\mu$  in an analysis of one and the same flight attitude, depends upon the choice of coupling between the body- and wind-axes systems. Its formal definition is, to be sure, independent of the coupling, but the position of the lift axis for a certain flight stage is different with the F-coupling ( $z_{aF}$ ) than with the E-coupling method ( $z_{aE}$ ). Angle  $\mu$  must then also carry a subscript ( $\mu_E$  or  $\mu_F$ ).

## VI. THE COUPLING BETWEEN AIRCRAFT AND GROUND SYSTEMS

In conformity with the previous arguments regarding the orientation of the individual flight-mechanic axes, the ground-axes system is chosen as orientating system. The principal axis of rotation of the ground system is again the perpendicular axis  $z_g$ , the position of which is known beforehand, while that of  $x_g$  and  $y_g$  is momentarily unknown and may be chosen arbitrarily.

Fundamentally, any of the three airplane axes could serve as definitive axis of rotation; but axis  $z$  proves little suitable because in one of the common positions of the airplane with respect to the ground axis,  $z$  and  $z_g$  coincide, for which the related Cardan model shows parallel rings in the neutral setting.

There remain, therefore, axes  $x$  and  $y$ . Orientation

of the airplane with respect to the ground by three Euler angles, one of which has axis  $y$  of the airplane as axis of rotation, can be effected immediately. In the described orientation of the aircraft axes relative to the axes  $x_a, y_a, z_a$  given by a ground plate and the flow, by the angles  $\alpha_E, \beta_E, \phi_E$  this coupling is practically applied. The choice of axis  $x$  as definitive axis of rotation, has the advantage of making the coupling angles between aircraft system and ground system amenable to definition in the same manner as between wind and ground systems; for in both cases corresponding axes in agreement in neutral position are definitive axes of rotation of the system to be orientated. These are  $x$  and  $x_a$ .

The three coupling angles  $\phi, \delta, \psi$  between aircraft system and ground system are, based upon axis  $x$  as definitive axis of rotation of  $x, y, z$ , and of axis  $z_g$  as principal axis of rotation  $x_g, y_g, z_g$ , explained as follows:

- $\delta$  angle of pitch - angle between  $x$  and plane  $x_g y_g$ . Axis of rotation of the angle is axis  $y$  projected on horizontal plane  $x_g y_g$ ; i.e., a horizontal axis at right angles to  $x$ , whose positive direction viewed along positive axis  $x$  is clockwise. Plane  $x z_g$  is the plane of measurement.
- $\phi$  angle of roll - angle between  $z$  and plane  $x z_g$ . Axis of rotation of the angle is formed by axis  $x$ . Plane  $y z$  is plane of measurement.
- $\psi$  azimuth angle - angle between  $x$  projected on plane  $x_g y_g$  and axis  $x_g$ . Axis  $z_g$  is axis of rotation of the angle. Plane  $x_g y_g$  is plane of measurement.

The definitions of the angles with the corresponding coupling angles between the wind and ground systems, are in agreement. Simply replace  $x_a$  by  $x$ ,  $y_a$  by  $y$  and  $z_a$  by  $z$  (fig. 13).

The following relations hold true:



From triangle  $x F x_g$ : 1.  $\cos(x x_g) = \cos \theta \cos \psi$

From triangle  $x F y_g$ : 2.  $\cos(x y_g) = \cos \theta \sin \psi$

$$3. \cos(x z_g) = -\sin \theta$$

From triangle  $y K x_g$ : 4.  $\cos(y x_g) = -\sin \psi \cos \varphi$   
 $+ \cos \psi \sin \varphi \sin \theta$

From triangle  $y K y_g$ : 5.  $\cos(y y_g) = \cos \psi \cos \varphi$   
 $+ \sin \psi \sin \varphi \sin \theta$

From triangle  $y x z_g$ : 6.  $\cos(y z_g) = \cos \theta \sin \varphi$

From triangle  $z K x_g$ : 7.  $\cos(z x_g) = \sin \varphi \sin \psi$   
 $+ \cos \varphi \cos \psi \sin \theta$

From triangle  $z K y_g$ : 8.  $\cos(z y_g) = -\sin \varphi \cos \psi$   
 $+ \cos \varphi \sin \psi \sin \theta$

From triangle  $z x z_g$ : 9.  $\cos(z z_g) = \cos \theta \cos \varphi$

The transfer from aircraft system  $x, y, z$  to ground system  $x_g, y_g, z_g$  has the form shown in:

TABLE XI

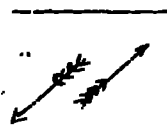
		Ground Axes		
		$x_g$	$y_g$	$z_g$
Body axes	$x$	$+\cos \theta \cos \psi$	$+\cos \theta \sin \psi$	$-\sin \theta$
	$y$	$+\sin \theta \cos \psi \sin \varphi$ $-\sin \psi \cos \varphi$	$+\sin \theta \sin \psi \sin \varphi$ $+\cos \psi \cos \varphi$	$+\cos \theta \sin \varphi$
	$z$	$+\sin \theta \cos \psi \cos \varphi$ $+\sin \psi \sin \varphi$	$+\sin \theta \sin \psi \cos \varphi$ $-\cos \psi \sin \varphi$	$+\cos \theta \cos \varphi$

Figure 14 gives the corresponding Cardan model of the coupling between aircraft and ground systems. The coupling angles  $\varphi, \theta, \psi$  are set positive.

## VII. GROUPING OF THE MOST SUITABLE AXES AND COUPLING ANGLES IN A "STANDARD" SYSTEM

In the selection of practical systems of axes, by means of which flight-mechanic problems are appropriately treated, and by the grouping of these axes and angles into a "standard" system, a number of subscripts necessary in the present report, for obvious reasons, can be eliminated. Following the ultimate choice of a certain coupling between aircraft and wind systems, namely, of the E coupling, all subscripts in capital letters (E, F, K) are eliminated. All subscripts in small letters are retained.

On the basis of the results of the investigation, the following standard system of axes and coupling angles of flight mechanics, can be formulated:

### A. Axes\*

1. Aircraft axes  $x, y, z$
2. Experimental axes  $x_o, y_o, z_o$
- 3a. Wind axes  $x_a, y_a, z_a^*$
- 3b. Tunnel axes  $x_a, y_a, z_a^*$
4. Ground axes  $x_g, y_g, z_g$

The axes cited under 1, 3a, and 4, are explained in detail in references 9 and 10.

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\*With the choice of the E coupling between aircraft and wind axes and tunnel axes, respectively, axis  $y_a$  and  $z_a$  of the wind system agree with axis  $y_{aK}$  and  $z_{aK}$  of the tunnel system. Axes  $x_a$  and  $x_{aK}$  are always identical (reference 9), regardless of the choice of coupling. For measurements on airplanes or models over a ground plate, axes  $x_{ap}, y_{ap}, z_{ap}$ , given by the axis of flow, and the ground plate are used, or else ground axes  $x_g, y_g, z_g$  are used, and because of the presence of the ground plate may be considered as given.

## B. Coupling Angles

1. Aircraft axes - wind axes, coupling through aerodynamic angles  $\alpha$  and  $\beta$ , which form the angles of the E coupling.
2. Aircraft axes - experimental axes, coupled through angle of attack  $\alpha$ .
3. Wind axes - experimental axes, coupled through angle of yaw  $\beta$ .
4. Wind axes - ground axes, coupled through angles  $\mu$ ,  $\gamma$ ,  $\chi$ .
5. Airplane axes - ground axes, coupling through angles  $\varphi$ ,  $\theta$ ,  $\psi$ .

Between the individual coupling angles, with E coupling as basis, the following relations exist between aircraft and wind axes.\*

- I.  $\cos \theta \sin \sigma = \sin \alpha \sin \mu + \cos \alpha \cos \mu \sin \beta$
- II.  $\cos \beta \cos \mu = \cos \varphi \cos \sigma + \sin \varphi \sin \sigma \sin \theta$
- III.  $\cos \gamma \cos \mu = \sin \theta \sin \alpha + \cos \theta \cos \alpha \cos \varphi$
- IV.  $\cos \gamma \sin \sigma = \sin \beta \cos \varphi + \cos \beta \sin \varphi \sin \alpha$
- V.  $\sin \varphi \cos \alpha = \sin \mu \cos \sigma + \cos \mu \sin \sigma \sin \gamma$
- VI.  $\cos \theta \sin \varphi = \sin \beta \sin \gamma + \cos \beta \cos \gamma \sin \mu$
- VII.  $\cos \alpha \cos \beta = \sin \theta \sin \gamma + \cos \theta \cos \gamma \cos \sigma$

In these equations,  $\sigma$  signifies the difference angle  $\psi - \chi$ .

Figure 15 illustrates the orientation of the aircraft axes relative to the wind and ground axes, as well as the coupling angles between wind and ground axes.

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\*The detailed derivation of the relations between the coupling angles for the E coupling, as well as for the F coupling will be published in a future report.

The identification of the axes of the aircraft wind and ground systems by  $x, y, z$ , together with subscripts  $a$  and  $g$ , has been internationally standardized by the I.C.A.N. and by the International Federation of National Standardizing Associations (ISA). Axes  $x_e, y_e, z_e$  of the experimental system agree in definition and subscripts with the axes known from English-American literature as "wind axes."

The ISA Committee, 20, Aviation, selected the E coupling for aircraft and wind systems, while the "rules governing the use of international symbols and terms in aeronautical engineering" of the ICAN contain at present, the F coupling, although the ICAN itself is now contemplating introduction of the E coupling.

The definition and notation for  $\varphi, \theta, \psi$  is internationally standardized except that the prefixes differ in the various countries, depending on the choice of positive sense of direction of the reference axes.

Translation by J. Vanier,  
National Advisory Committee  
for Aeronautics.

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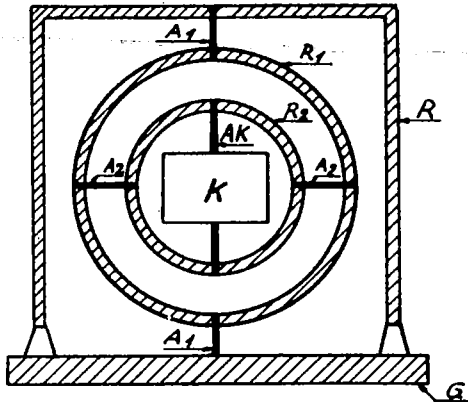


Figure 1.- Cardan model illustrating the Euler angles.

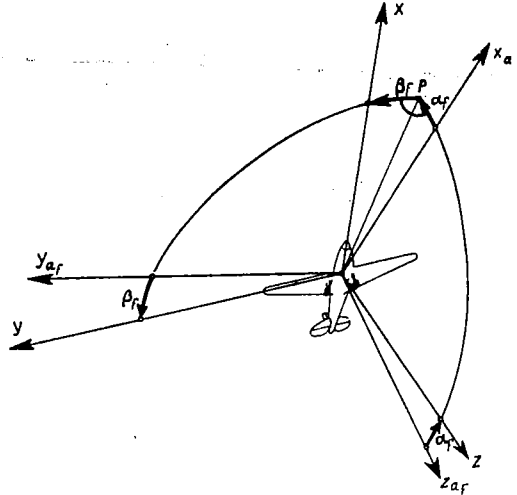


Figure 4.- Orientation of aircraft system  $x, y, z$  relative to wind system  $x_a, y_a, z_a$  by the aerodynamic angles  $\alpha_F$  and  $\beta_F$  of the F coupling.

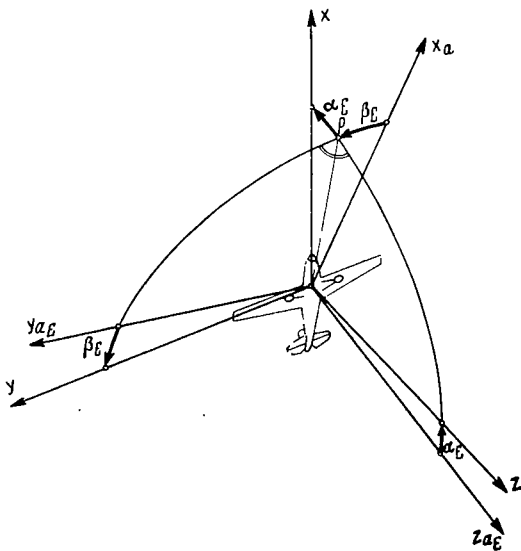


Figure 6.- Orientation of aircraft system  $x, y, z$  relative to wind system  $x_a, y_a, z_a$  by the aerodynamic angles  $\alpha_E$  and  $\beta_E$  of the E coupling.

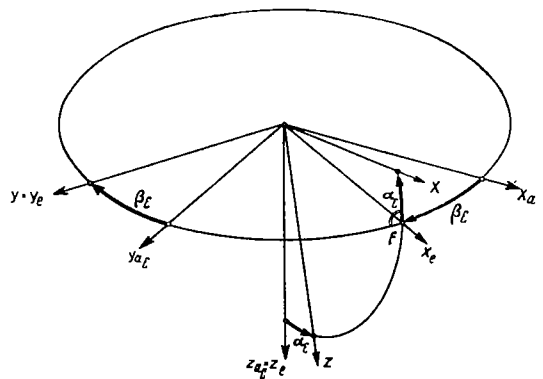


Figure 7.- Orientation of experimental system  $x_e, y_e, z_e$  relative to wind system  $x_a, y_a, z_a$  defined by the E coupling.

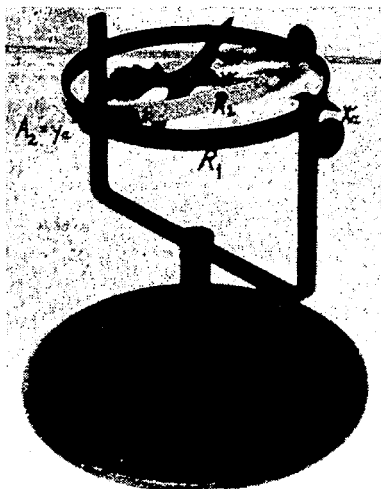


Figure 2.- Cardan model of coupling between aircraft system  $x,y,z$  and wind system  $x_a,y_a,z_a$  for the case where axis  $x$  is axis of rotation of the aircraft system.

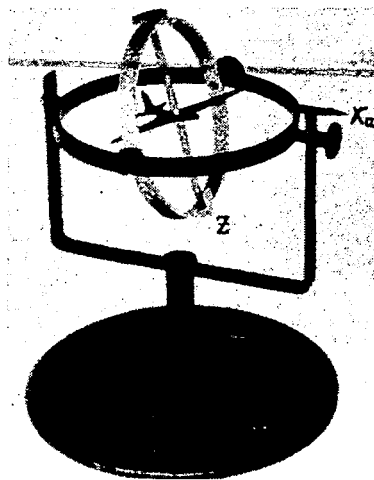


Figure 3.- Cardan model of the F coupling; angle of attack  $\alpha_F$  and angle of yaw  $\beta_F$  are set positive.

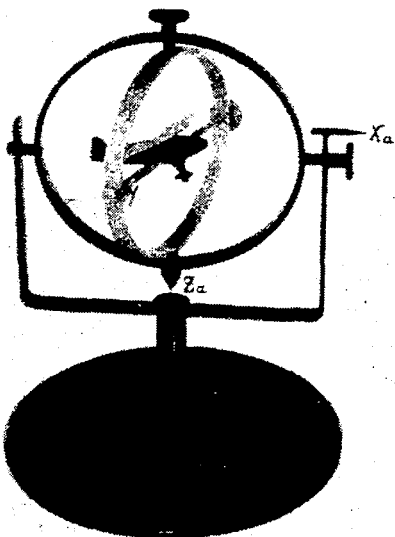


Figure 5.- Cardan model of the E coupling; angle of attack  $\alpha_E$  and angle of yaw  $\beta_E$  set positive.

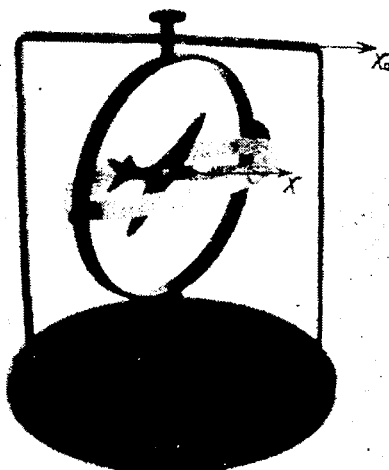


Figure 14.- Cardan model of coupling between aircraft system  $x,y,z$  and ground system  $x_g,y_g,z_g$ . Angles  $\phi, \psi, \psi$  are in positive setting.

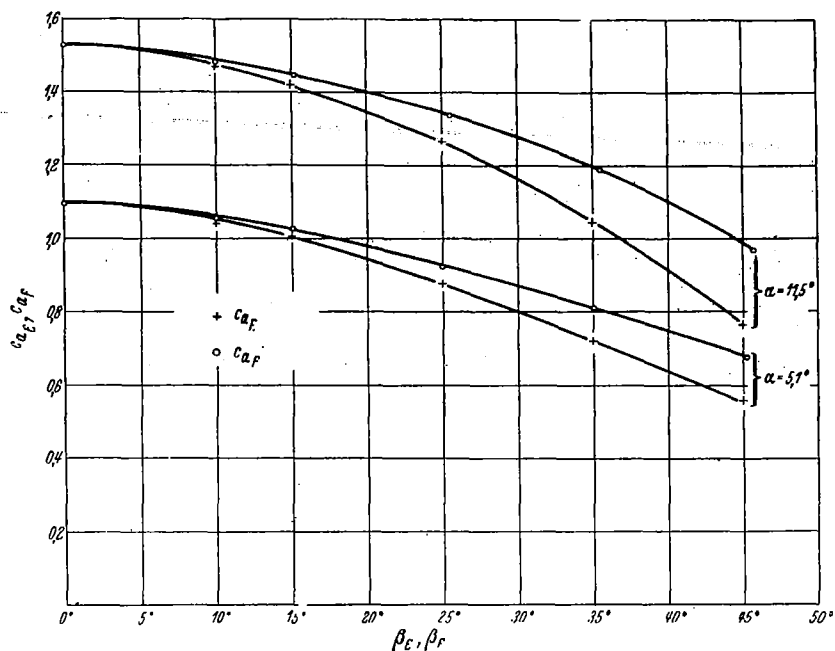


Figure 8.- Lift coefficient  $C_{aE}$  and  $C_{aF}$  against angle of yaw  $\beta_E$  or  $\beta_F$  on a high-wing land plane.

Figure 9.- Lift coefficients  $C_{aE}$  and  $C_{aF}$  plotted against angle of yaw  $\beta_E$  or  $\beta_F$  on a high-wing seaplane.

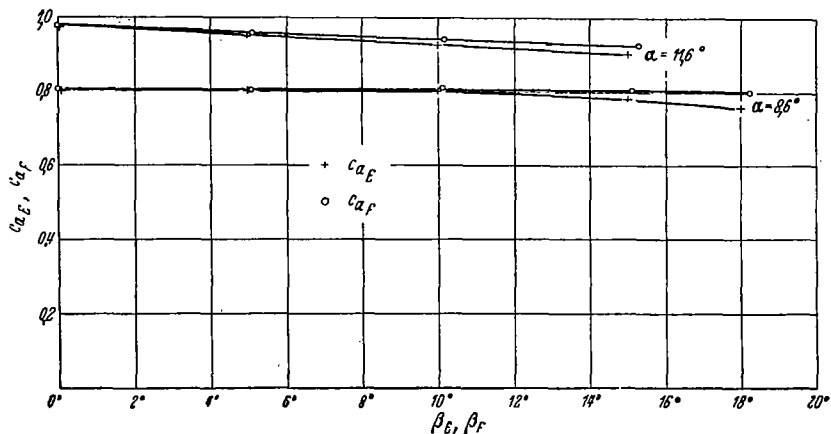
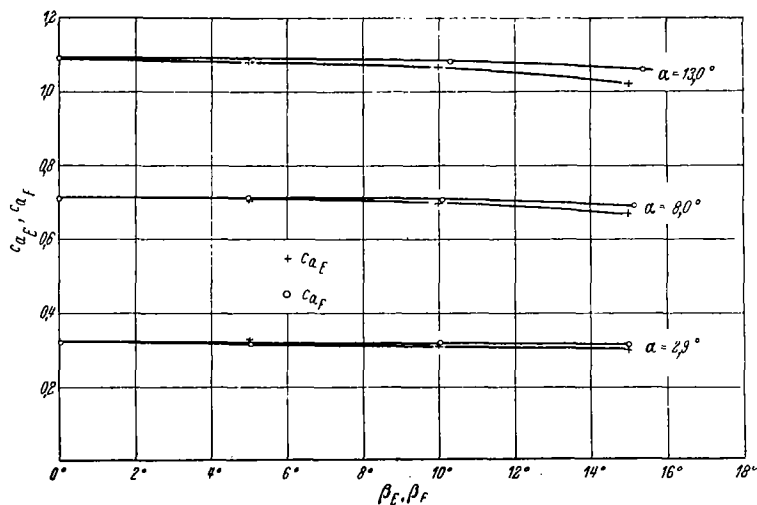


Figure 10.-  $C_{aE}$  or  $C_{aF}$  against  $\beta_E$  or  $\beta_F$  on a low-wing land plane.



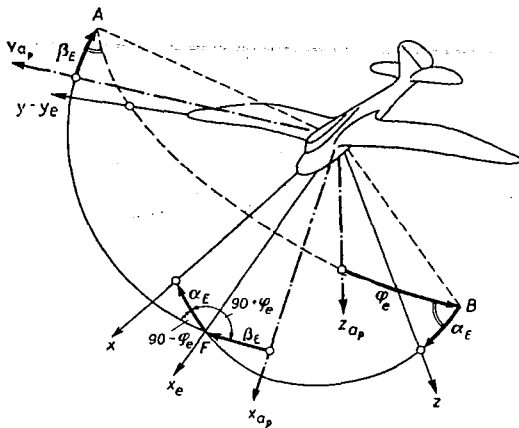


Figure 11.- Orientation of aircraft system  $x, y, z$  with respect to wind system  $x_{ap}, y_{ap}, z_{ap}$  for measurements with ground plate.

Figure 12.- Orientation of wind system  $x_a, y_a, z_a$  with respect to ground system  $x_g, y_g, z_g$ .

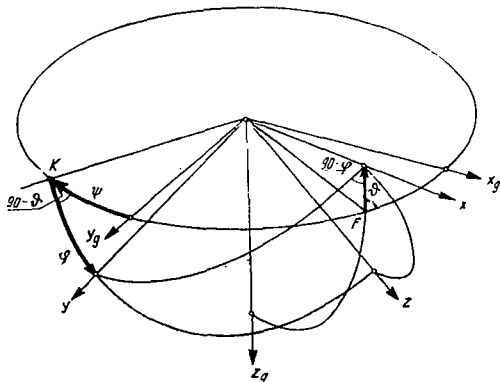
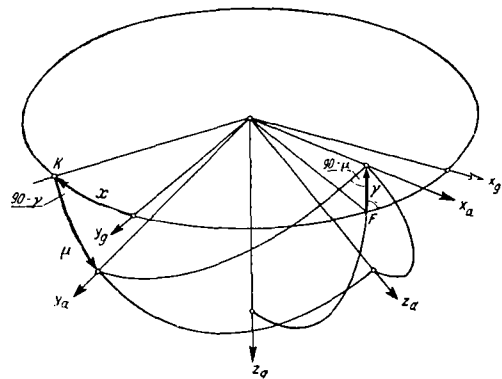
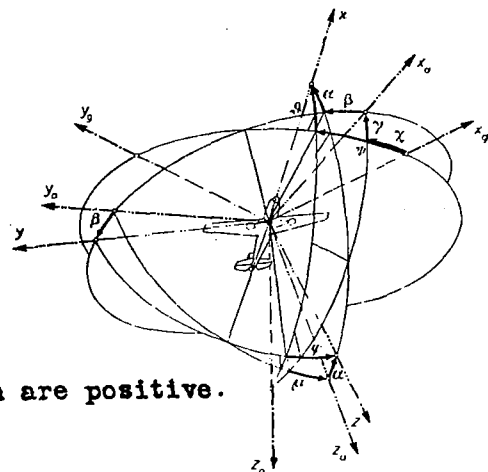


Figure 13.- Orientation of aircraft system  $x, y, z$  with respect to ground system  $x_g, y_g, z_g$ .

Figure 15.- Orientation of aircraft, wind and ground systems, based on E coupling.



All angles shown are positive.

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